Earthworks Planning for Road Construction Projects: A Case Study

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Abstract

In this paper we construct earthwork allocation plans for a linear infrastructure road project. Fuel consumption metrics and an innovative block partitioning and modelling approach are applied to reduce costs. 2D and 3D variants of the problem were compared to see what effect, if any, occurs on solution quality. 3D variants were also considered to see what additional complexities and difficulties occur. The numerical investigation shows a significant improvement and a reduction in fuel consumption as theorised. The proposed solutions differ considerably from plans that were constructed for a distance based metric as commonly used in other approaches. Under certain conditions, 3D problem instances can be solved optimally as 2D problems.

Keywords: mass-haul optimisation, earthworks allocation, fuel consumption, emissions

1. Introduction

In this paper we apply a new planning technique to a road construction case study. Road construction can be the source of very large and costly earthworks as many have significant length and pass through difficult terrain. In this type of linear infrastructure project the principal earthwork operations are: i) stripping vegetation and topsoil, ii) loosening material in cutting and borrow pits, iii) excavating material, iv) loading material from cuts and hauling to fills (or to spoil), v) spreading, shaping, watering, compacting and trimming the fill material (QTMR 1977). The main idea of earthworks is to alter an existing land surface into a desired configuration by excavating material from specific locations and using that material as fill in other locations. This earthwork problem is commonly referred to as mass-haul. A solution to this problem is an earthwork allocation plan (EAP) that describes where cut material is to be placed as fill and how much material is to be moved between each cut-fill pairing. There are two golden rules that must also be borne in mind during these projects; don't double handle material whenever possible and always load and carry material downhill (QTMR 1977).

In the last ten years much research has been performed in this field. Recent papers include Son et al. (2005), Aruga et al. (2005), Akay (2006), Karimi et al. (2007), Kim et al. (2007), Goktepe et al. (2008), Zhang (2008), Dawood and Castro (2009), Ji et al. (2010), Hola and Schabowicz (2010), Ji et al. (2011), Shah and Dawood (2011), Nassar and Hosney (2012). However, there are many limitations and inaccuracies in this work, and much of it is not comprehensive or detailed enough to be readily applicable to real life. A comprehensive review and critical analysis of this field by Burdett and Kozan (2013a) has recently shown

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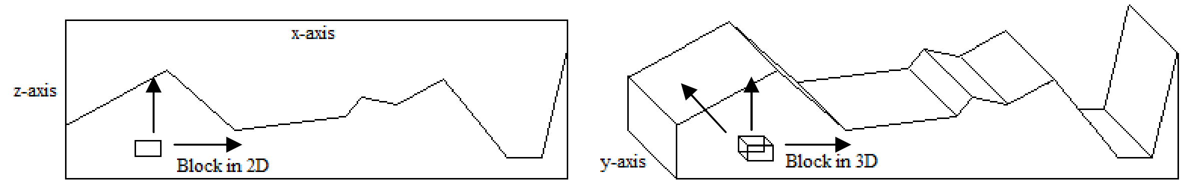
that there are many opportunities and avenues for future research, and that a variety of new approaches can be developed. Consequently in Burdett and Kozan (2013b) a number of new approaches were developed. The partitioning of the project site into blocks and the development of associated block models was the foremost approach in that paper. The block models are a superior approach to previous section based alternatives as "conceptually" they model more realistically and accurately the position of earth at different elevations. The block models are generic and are readily applicable for both 2D and 3D scenarios. Figure 1 in particular shows the difference between 2D and 3D scenarios. The path between blocks and other locations is not direct, i.e. it is the sum of separate movements over inclined planes of different angles. This approach is superior to those taken in other papers and models more realistically the movement of material through the terrain as it is altered.



a) Movements of earth across x and z axis ***b) Movements across x, y and z axis***

Figure 1. Comparison of 2D and 3D earthwork scenarios

In Burdett and Kozan (2013b) the physics concept "work" has also been used as a proxy for fuel consumption. It is a generic and robust metric for the movement of earth over inclined planes of different angles and surfaces. That approach is necessary as it is impractical to measure and quantify the exact fuel consumption of every construction vehicle and for every associated factor. Fuel consumption was used as a metric instead of haul distance or haul time for several reasons. Foremost is that distance travelled is not an accurate measure of cost as the gradient of travel (among other important factors) is not included. The gradient of travel significantly affects the ease or difficulty of moving material and greatly effects fuel consumption. Recently the environmental impact of construction activities has become an important topic due to the pollution that is created. In order to quantify emissions, the fuel consumption of vehicles performing hauls must be quantified. The reduction of fuel consumption though is important in its own right, as recent site visits to the Bruce Highway upgrade at Gympie (Queensland) has revealed that there is often hundreds of thousands of dollars worth of fuel sitting in vehicles on project sites.

2. A Review of Block Optimisation Models

This approach is based upon the partitioning of the problem domain into rectangular prisms (i.e. 3D blocks). Our previous paper Burdett and Kozan (2013b) should be consulted for extended details. The following is a review of the most pertinent features:

The set of blocks requiring excavation and fill respectively are denoted by (B^-, B^+). Borrow and waste site blocks are regarded as auxiliary blocks. They are members of set AUX. Borrow site blocks are however included in B^- as they involve excavations and are sources of earth for filling activities. Similarly waste site blocks are included in B^+ as they are locations for fill. The size of each block *b* is specified by $(\Delta x, \Delta y, \Delta z)$ and its position in three dimensional space is given by the grid location, (x_b, y_b, z_b). Its middle point is similarly, $m_b = ((x_b - 0.5)\Delta x, (y_b - 0.5)\Delta y, (z_b - 0.5)\Delta z)$. The volume of each block is denoted as v_b and the predominant soil type in each block is denoted as ω_b. However the volume of each type required (i.e. as cut and fill) is denoted by $cut_{b,s}$ and $fill_{b,s}$ respectively. A binary parameter that equals one if material of type *s* is acceptable in block *b*, and zero if material is not acceptable, is defined as $\kappa_{b,s}$. The fuel consumption of vehicles making hauls between block *b* and *b*' is denoted by $f_{b,b'}$. A binary decision variable denoted by $H_{b,b'}$ is defined to signify whether material is hauled from block *b* to b'. The volume to be cut from block *b* and moved to block b' of soil type *s* is also denoted as $Q_{b,b',s}$. Two competing block optimisation models utilise these decision variables. They are shown below.

Block Model BLM-1 (Mixed Integer Programming (MIP)):

Minimise $\sum_{b \in B^-} \sum_{b' \in B^+} (f_{b,b'} \times H_{b,b'} \times v_b / \hat{Q}_{b,b'})$ [Total fuel consumption]

Subject to:

$\sum_{b' \in B^+} (H_{b,b'}) = 1 \quad \forall b \in B^- | b \notin AUX$ [Blocks moved to one destination]

$\sum_{b' \in B^-} (H_{b',b}) = 1 \quad \forall b \in B^+ | b \notin AUX$ [Blocks filled from one source]

$\sum_{b' \in B^+} (H_{b,b'}) \leq 1 \quad \forall b \in B^- | b \in AUX$ [Borrow site utilisation]

$\sum_{b' \in B^-} (H_{b',b}) \leq 1 \quad \forall b \in B^+ | b \in AUX$ [Waste site utilisation]

$H_{b,b'} = 0 \quad \forall b \in B^-, \forall b' \in B^+ | \omega_b \neq \omega_{b'}$ [Haulage restriction]

$H_{b,b'} \leq \kappa_{b',\omega_b} \quad \forall b \in B^-, \forall b' \in B^+$ [Haulage restriction]

$H_{b,b'} = 0 \quad \forall b \in B^- | b \in AUX, \forall b' \in B^+ | b' \in AUX$ [Borrow to waste site restriction]

$H_{b,b'} \in \{0,1\} \quad \forall b \in B^-, \forall b' \in B^+$ [Binary constraint]

Block Model BLM-2 (Linear Programming (LP)):

Minimise $\sum_{b \in B^-} \sum_{b' \in B^+} \sum_s (f_{b,b'} \times Q_{b,b',s} / \hat{Q}_{b,b',s})$ [Total fuel consumption]

Subject to:

$\sum_{b' \in (B^+)} (Q_{b,b',s}) = cut_{b,s} \quad \forall b \in B^- | b \notin AUX, \forall s$ [Cutting constraint]

$\sum_{b' \in (B^-)} (Q_{b',b,s}) = fill_{b,s} \quad \forall b \in B^+ | b \notin AUX, \forall s$ [Filling constraint]

$\sum_{b' \in (B^+)} (Q_{b,b',s}) \leq cut_{b,s} \quad \forall b \in B^- | b \in AUX, \forall s$ [Borrow site utilisation]

$\sum_{b' \in (B^-)} (Q_{b',b,s}) \leq fill_{b,s} \quad \forall b \in B^+ | b \in AUX, \forall s$ [Waste site utilisation]

$Q_{b,b',s} \geq 0 \quad \forall s, \forall b \in B^-, \forall b' \in B^+$ [Positivity restriction]

$Q_{b,b',s} = 0 \quad \forall s, \forall b \in B^-, \forall b' \in B^+ | b, b' \in AUX$ [Borrow to waste site restriction]

$Q_{b,b',s} = 0 \quad \forall s, \forall b \in B^- | cut_{b,s} = 0, \forall b' \in B^+ | fill_{b',s} = 0$ [Haulage restriction]

These models differ quite considerably from previous approaches that only model the movement of material between sections in a horizontal fashion. Hence they are conceptually superior and somewhat novel. The two models are slightly different and it is a matter of conjecture (at present) which is most applicable to industry. The first model considers where blocks are to be moved to as fill. It is assumed that soil type within each block is uniform and blocks are whole. That is each cut block is initially full of material and each fill block has no material. Therefore "whole" blocks are cut, moved, and placed as fill and a blocks material is not divided; it goes to one location. This is not an unrealistic assumption in many situations. From a practical perspective it is perhaps more realistic to dig up a discrete block of earth and to shift it to one specified place as opposed to trying to accurately break it up into many smaller parts and to send them to many specific locations. The second model is a relaxation of the first. Therefore block material can be divided and hauled as fill to many different locations. Conceptually the second model is superior, but practically it may not be possible to implement a solution as exactly specified by this model. It should be noted that $\hat{Q}_{b,b}$ and $\hat{Q}_{b,b',s}$ denote the capacity of the vehicle making hauls between blocks. Dividing by these values, in the objective function, computes the number of hauls to be made. Details concerning the calculation of fuel consumption and other parameters can be found in Burdett and Kozan (2013b).

3. Case Study

The case study is a road construction project from Northern Queensland (Australia). Its length is approximately 7 km. The terrain and planned road profiles are shown in Figure 2.

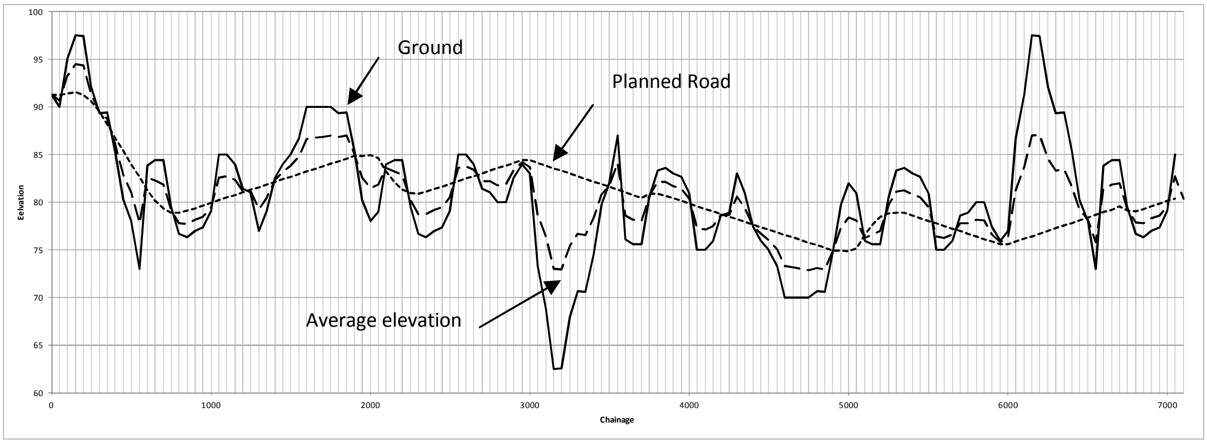


Figure 2. Longitudinal profiles of ground and planned road surface x-axis (chainage): 0-7 km; y-axis: 60 – 100 m (elevation)

The land is assumed to follow the line connecting adjacent elevations. Data occurs every 50 metres and the road width is 20 metres. A significant amount of cutting and filling are required and these "raw" amounts are fairly well balanced. The earthwork volumes are based purely on the longitudinal profiles, i.e. as the difference between the blue and red lines in Figure 2. The net cut and fill required is 304231.85 and 299246.81 cubic metres respectively; a difference of 4985 cubic metres. For this case study, one soil type has been considered and the cost of fuel is $1.50 per litre. Waste sites are placed at either end of the project (i.e. one for each cross section) and one borrow site occurs 30km to the right of the project site.

This case study has already been partially considered in Burdett and Kozan (2013b) but only 2D variants were solved in that paper and multiple soil types were addressed. A section based approach was also applied in that paper. It was found to be inferior, and therefore has not been reconsidered in this paper. As most construction projects are 3 dimensional, this paper provides an important link between theory and practice.

4. Numerical Investigations

This numerical investigation concentrates on solving 3D variants of the case study described in the preceding section. This is done as many road projects are solved as 2D problems. However some 2D variants are also solved for comparative purposes. A 3D problem is simply one that partitions the domain in each axis. Previous 2D variants of the problem did not partition the project site across the y-axis (i.e. only in x-z axis). In other words there was only a single cross section containing blocks.

Various blocks sizes have been investigated. This size directly affects the total number of blocks and is limited by the available memory on a computer. As blocks get smaller, more are required, and more decisions need to be made. From a theoretical viewpoint, the solution of earthwork problems with greater number of blocks is highly significant and very challenging from a computational point of view. However from a purely practical viewpoint, block sizes below a certain size are of limited value (at least in the near future) to contractors who perform the earthworks.

In this paper the OPL Studio software (also known as CPLEX) has been used to solve the mathematical “block” models. A quad core, Dell PC with a 2.5 GHz processor and 4GB memory has been used. The model parameters were computed in C++.

4.1. Cross Section Replication

Provided that elevations are constant across the y-axis, it is theorised that an optimal solution to the full 3D problem could be obtained by replicating the answer for a single 2D slice. In other words there is no movement of material between cross sections. This approach significantly reduces the size of the problem that must be solved. It also means that smaller blocks can be used and more blocks can be used within a single cross section. Otherwise, blocks have to be distributed across all three dimensions and fewer blocks can be used per cross section. This idea is shown in Figure 3.

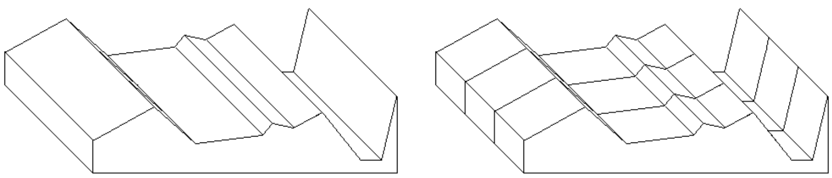


Figure 3. Single cross section versus multiple (i.e. three)

For this approach, the blocks models were applied and the results are shown Table 1 and 2.

Table 1. BLM-1 results for cross section replication approach

Variant	Block Size (sx, sy, sz)	#Block	#Cross Section	Metric	Distance (km)	Work (J)	Fuel Cons (litres)	Cost ($)	Cost Diff. ($)
2D	(100, 20, 1)	582	× 1	Work	31,115	9.5581E+10	95,581	143,372	79,243
				Distance	29,467	1.4841E+11	148,410	222,615	
2D	(100, 10, 1)	582	× 2	Work	15,557	4.7791E+10	47,791	71,686	79,244
				Distance	14,734	7.4205E+10	74,205	111,308	
2D	(100, 5, 1)	582	× 4	Work	7,779	2.3895E+10	23,895	35,843	79,244
				Distance	7,367	3.7102E+10	37,102	55,654	
2D	(50, 20, 1)	953	× 1	Work	21,071	6.6134E+10	66,134	99,201	34,575
				Distance	19,509	8.9184E+10	89,184	133,776	

2D	(50, 10, 1)	953	× 2	Work	10,535	3.3067E+10	33,067	49,600	34,576
				Distance	9,755	4.4592E+10	44,592	66,888	
2D	(50, 5, 1)	953	× 4	Work	5,268	1.6533E+10	16,533	24,800	34,576
				Distance	4,877	2.2296E+10	22,296	33,444	

Table 2. BLM-2 results for cross section replication approach

Variant	Block Size (sx, sy, sz)	#Block	#Cross Section	Metric	Distance (km)	Work (J)	Fuel Cons (litres)	Cost ($)	Cost Diff. ($)
2D	(100, 20, 1)	582	× 1	Work	13,776	4.9518E+10	49,518	74,277	15,934
				Distance	12,621	6.0141E+10	60,141	90,211	
2D	(100, 10, 1)	582	× 2	Work	6,888	2.4759E+10	24,759	37,138	15,936
				Distance	6,311	3.0070E+10	30,070	45,106	
2D	(100, 5, 1)	582	× 4	Work	3,444	1.238E+10	12,379	18,569	15,936
				Distance	3,155	1.5035E+10	15,035	22,553	
2D	(50, 20, 1)	582	× 1	Work	13,856	5.4556E+10	54,556	81,834	10,475
				Distance	12,651	6.1540E+10	61,540	92,309	
2D	(50, 10, 1)	953	× 2	Work	6,928	2.7278E+10	27,278	40,917	10,476
				Distance	6,326	3.0770E+10	30,770	46,155	
2D	(50, 5, 1)	953	× 4	Work	3,464	1.3639E+10	13,639	20,459	10,472
				Distance	3,163	1.5385E+10	15,385	23,077	

The distance metric used was Euclidean distance. The cross section column reports the number of cross sections and that the solution to the full 3D problem is the specified number of times larger. Table 1 and 2 shows that the work metric provides a reasonable improvement over those solutions obtained with the distance metric. The solutions obtained for a given (sx, sz) are all the same and different sy values had no effect on the final solution. The cost difference between BLM-1 is greater than BLM-2. This occurs because BLM-1 has less flexibility in how earth is to be divided. For example, a blocks worth of earth can only be moved to one destination. BLM-2 is given the opportunity to divide the earth as it sees fit.

4.2. 3D Instances

In this section, the y-axis is fully partitioned into blocks, and movement between cross sections is not limited. The results are shown in Table 3 and 4.

Table 3. BLM – 1 results for standard 3D problems

Variant	Block Size (sx, sy, sz)	#Blocks	Metric	Distance (km)	Work (J)	Fuel Cons (litres)	Cost ($)	Cost Diff. ($)
3D	(100, 10, 1)	1164	Work	31,115	9.5581E+10	95,581	143,372	79,243
			Distance	29,467	1.4841E+11	148,410	222,615	
3D	(100, 5, 1)	2328	Work	31,115	9.558E+10	95,581	143,372	79,243
			Distance	29,467	1.484E+11	148,410	222,615	
3D	(50, 10, 1)	1906	Work	21,071	6.6134E+10	66,134	99,201	34,575
			Distance	19,509	8.9184E+10	89,184	133,776	
3D	(50, 5, 1)	3812	Work	-	-	-	-	
			Distance	-	-	-	-	

Table 4. BLM – 2 results for standard 3D problems

Variant	Block Size (sx, sy, sz)	#Blocks	Metric	Distance (km)	Work (J)	Fuel Cons (litres)	Cost ($)	Cost Diff. ($)
3D	(100, 10, 1)	1164	Work	13,776	4.9518E+10	49,518	74,277	15,934
			Distance	12,621	6.0141E+10	60,141	90,211	
3D	(100, 5, 1)	2328	Work	13,776	4.9518E+10	49,518	74,277	15,934

			Distance	12,621	6.0141E+10	60,141	90,211	
3D	(50, 10, 1)	1906	Work	13,856	5.4556E+10	54,555	81,832	10,485
			Distance	12,651	6.1540E+10	61,544	92,317	
3D	(50, 5, 1)	3812	Work	-	-	-	-	-
			Distance	-	-	-	-	

The time to solve the models was no more than several minutes in the majority of cases. On one of the largest problem instances (i.e. for the binary decision model BLM-1), the CPU time was in the vicinity of fifteen minutes. The last test problem in Tables 3 and 4 could not be solved as insufficient memory was available to generate the model. These tables show partitioning across the y-axis had no effect on the solution. The results are also the same as those obtained in Table 1 and 2. Therefore when elevations are constant across the y-axis, we can conclude that a 3D problem is optimally solved as a 2D problem. Upon closer investigation the reason for this is that distance is a significant component of fuel consumption and the distance between two blocks is shorter if the blocks are in the same cross section. This is similarly true as the elevations are the same across the y-axis. Graphically this is shown in Figure 4. For example it is less costly to move material from (1,1) to (2,1) as the distance is shorter. As the elevation at (2,2) and (2,3) are the same as (2,1), then there is no change in gradient.

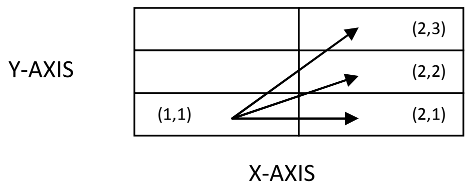


Figure 4. Increased distance when hauling between block (1,1) and (2,2) or (2,3).

Even though obtained solutions are the same in terms of cost and fuel consumption, we should point out that the solutions (i.e. the allocations) are all different. Therefore the numerical investigations as reported in these tables are still beneficial and required. These tables also show that each instance is actually solvable - something we would not otherwise know.

4.3. Revised 3D Instances

Because of the results of the preceding sub section, larger problems can be simplified and solved by cross section replication. If any of the aforementioned conditions are violated then we believe it is possible to improve the solution by partitioning in 3D. To test this hypothesis we have altered the position of waste (Wa) and borrow (Bo) sites and have resolved the optimisation models. The new position of waste and borrow sites is shown in Figure 5.

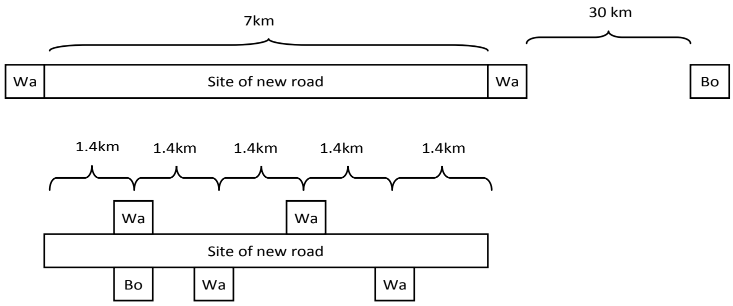


Figure 5. Original layout and the altered layout

The borrow site is now positioned closer to the road and is a more viable source of earth. Other material must therefore be disposed of in waste sites which are placed equidistantly along the length of the new road. The new results are shown in Table 5 and 6. These results show that the solution differs when the problem is partitioned into two or four cross sections. Hence the results of the previous section are in fact caused by constant elevations across the y-axis and certain symmetries in the position of borrow and waste sites. The total work required was negative in some solutions, and this means that the net effect of earth movements is downhill travel. This result in a rarity and is only obtainable because borrow and waste sites are very close to the project site, i.e. they are equidistantly placed on either side. This result is useful as it demonstrates that such an arrangement is highly beneficial and ensures that fuel consumption is minimal.

Given the different results obtained in Table 5 and 6, an interesting question that comes to mind is, how good is the cross section replication approach for solving the revised 3D problems? In other words, how close to optimal are the solutions? To answer this question the cross section replication approach was applied to the revised case study. The results are shown in Table 7 and 8. The values within brackets are for the whole problem and not for a single cross section.

Table 5. BLM – 1 results for revised 3D problems

Variant	Block Size (sx, sy, sz)	#Blocks	Metric	Distance (km)	Work (J)
3D	(100, 10, 1)	1164	Work	18,994	-1.6559E+10
			Distance	17,811	3.4148E+10
3D	(100, 5, 1)	2328	Work	19,630	-1.1172E+10
			Distance	18,147	3.7988E+10
3D	(50, 10, 1)	1906	Work	17,152	2.9940E+10
			Distance	15,772	5.5725E+10

Table 6. BLM – 2 results for revised 3D problems

Variant	Block Size (sx, sy, sz)	#Blocks	Metric	Distance (km)	Work (J)
3D	(100, 10, 1)	1164	Work	12,927	4.0608E+10
			Distance	12,031	5.2135E+10
3D	(100, 5, 1)	2328	Work	12,856	3.9993E+10
			Distance	11,959	5.1522E+10
3D	(50, 10, 1)	1906	Work	12,906	4.5116E+10
			Distance	12,019	5.4058E+10

Table 7. BLM – 1 results for cross section replication approach (revised 3D problems)

Variant	Block Size (sx, sy, sz)	#Blocks	#Cross Section	Metric	Distance (km)	Work (J)
2D	(100, 20, 1)	582	×1	Work	18,660	-2.2945E+10
				Distance	17,407	2.5685E+10
2D	(100, 10, 1)	582	×2	Work	9,328 (18,656)	-1.1491E+10 (-2.2982E+10)
				Distance	8,701 (17,402)	1.2823E+10 (2.5645E+10)
2D	(100, 5, 1)	582	×4	Work	4,664 (18,654)	-5.7490E+09 (-2.2996E+10)
				Distance	4,350 (17,400)	6.4075E+09 (2.5630E+10)

Table 8. BLM – 2 results for cross section replication approach (revised 3D problems)

Variant	Block Size (sx, sy, sz)	#Blocks	#Cross Section	Metric	Distance (km)	Work (J)
2D	(100, 20, 1)	582	×1	Work	12,928	4.0613E+10
				Distance	12,032	5.2150E+10
2D	(100, 10, 1)	582	×2	Work	6,432 (12,864)	1.9016E+10 (3.8032E+10)
				Distance	5,962 (11,923)	2.4695E+10 (4.9389E+10)
2D	(100, 5, 1)	582	×4	Work	3,216 (12,863)	9.5070E+09 (3.8028E+10)
				Distance	2,981 (11, 923)	1.2346E+10 (4.9385E+10)

These results are quite interesting as they show that the solution is steadily improving as the width of each cross section gets smaller and the number of cross sections increases. In relation to the results presented in Table 5 and 6, the above results are difficult to interpret. In an attempt to make more sense of the results, Table 9 and 10 have been provided. All that can really be concluded however is that the results are comparable. It is unclear why the cross section replication approach has returned slightly better solutions on a number of occasions.

Table 9. BLM-1 result comparisons

Variant	Block Size	Work	Fuel Cons	Cost
3D	(100,10,1)	-1.655900E+10	-16,559	-24,838
3D	(100,5,1	-1.117200E+10	-11,172	-16,758
2D×1	(100,20,1	-2.294500E+10	-22,945	-34,417
2D×2	(100,10,1	-2.298200E+10	-22,982	-34,473
2D×4	(100,5,1)	-2.299600E+10	-22,996	-34,494

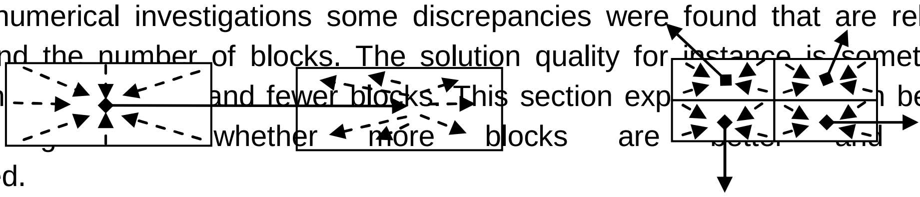
Table 10. BLM-2 result comparisons

Variant	Block Size	Work	Fuel Cons	Cost
3D	(100,10,1)	4.060800E+10	40,608	60,912
3D	(100,5,1)	3.999300E+10	39,993	59,989
2D×1	(100,20,1)	4.061300E+10	40,613	60,919
2D×2	(100,10,1)	3.803200E+10	38,032	57,048
2D×4	(100,5,1)	3.802800E+10	38,028	57,042

We conclude that the cross section replication approach is a viable approach to solving many problems. This approach is particularly useful for large problems that cannot be generated conventionally. If the cut and fill amounts are quite different across the y-axis then replicating the solution for a single cross section will not be useful. In this situation, each cross section can be solved separately. In this way the whole problem can be considered and a reasonable solution (but not optimal) can be obtained.

5. Block Size Benefits and Anomalies

In the numerical investigations some discrepancies were found that are related to block sizes and the number of blocks. The solution quality for instance is sometimes superior when th[illegible] and fewer blocks. This section exp[illegible] behind this and sheds [illegible] whether more blocks are better and should be modelled.



a) Haulage between large blocks ***b) Division into smaller sub blocks***

Figure 6. Effect of block size on earth movements

It has been assumed that all hauls occur from the middle point of one block to the middle point of another (see Figure 6a). This is a common practice and has been used in previous section based modelling approaches. All the movements required to haul material to the blocks central position (i.e. dotted red arrow in Figure 6) are however ignored in the current calculations. When the block sizes are decreased (see Figure 6b), then less material must be moved to the blocks centre (i.e. as they are smaller). In comparison to larger blocks, more of the earth movements are then taken into account and are modelled explicitly in the block optimisation models. Because more movements are modelled, the solution can be worse than the solution produced for larger block sizes. Though they have inferior key performance indicators (kpi), they are really better.

To prove this analytically, we define the effort of moving material to the centre of each block as follows:

$$\mathbf{TKM}(vol, dx, dy, nx, ny) = amt \times \sum_{i=1}^{nx} \sum_{j=1}^{ny} \sqrt{\left(x_i^{\text{mid}} - x^{\text{mid}}\right)^2 + \left(y_j^{\text{mid}} - y^{\text{mid}}\right)^2}$$
$$amt = \frac{vol}{nx \times ny}, \quad sx = dx/nx, \; sy = dy/ny,$$
$$x^{\text{mid}} = 0.5dx, \quad y^{\text{mid}} = 0.5dy,$$
$$x_i^{\text{mid}} = (i - 0.5) \times sx, \quad y_j^{\text{mid}} = (j - 0.5) \times sy$$

In the above equations TKM stands for tonne-kilometres. It is the sum of the product of the amount hauled by the haul distance. The equation for computing this amount is based upon dividing the block (which is of size (dx, dy)) into a number of sub blocks. The number of these in each axis is specified by *nx* and *ny* respectively. Hence the size of each sub block is given by (sx, sy). The movement from the middle of each sub block to the blocks middle point is then aggregated. The Euclidean distance is utilised. The amount of material within each sub block is equal, and given by *amt*. This equation can also be easily extended for 3D situations.

As the number of sub blocks increases the TKM value becomes more accurate and includes more and more of the internal movements. This is shown in Table 11 for a square block. It should be noted that nx and ny can be different. For any two values, it does not matter which is defined as nx or ny provided that the original block is a square (see Table 12). If the block is not square then the choice of nx and ny have a different effect (see Table

13 and Table 14). The value of TKM also does not necessarily increase with the number blocks. These differences all affect the solution of an earthwork allocation plan.

Table 11. TKM values for a 100x100 block with 500 tonnes of material

nx	ny	#blk	TKM	nx	ny	#blk	TKM
2	2	4	17677.7	8	8	64	19021.5
3	3	9	17883.1	9	9	81	19023.4
4	4	16	18721	10	10	100	19059.7
5	5	25	18743.6	20	20	400	19111.9
6	6	36	18940.9	100	100	10000	19129.2
7	7	49	18946.4	1000	1000	1000000	19129.9

Table 12. More TKM values for a 100x100 block with 500 tonnes of material

nx	ny	#blk	TKM	nx	ny	#blk	TKM
2	1	2	12500	7	1	7	12244.9
1	2	2	12500	1	7	7	12244.9
4	1	4	12500	3	2	6	18055.6
1	4	4	12500	2	3	6	18055.6

Table 13. TKM values for a 100x20 block with 500 tonnes of material

nx	ny	#blk	TKM	nx	ny	#blk	TKM
2	2	4	12747.5	8	8	64	12974.4
3	3	9	11998.5	9	9	81	12944.2
4	4	16	12893.9	10	10	100	12989.3
5	5	25	12714.3	20	20	400	13013.3
6	6	36	12947.9	100	100	10000	13022.7
7	7	49	12882.4	1000	1000	1000000	13023.1

Table 14. More TKM values for a 100x20 block with 500 tonnes of material

nx	ny	#blk	TKM	nx	ny	#blk	TKM
2	1	2	12500	7	1	7	12244.9
1	2	2	2500	1	7	7	2448.98
4	1	4	12500	3	2	6	12068.7
1	4	4	2500	2	3	6	12791.2

Given the effect of moving earth within blocks, this aspect should be included in all earthwork allocation models. Therefore all fuel consumption costs $f_{b,b'}$ should also include the fuel consumption of moving earth to the centre of block b and moving material away from the centre of block b'. The following example demonstrates the effectiveness of dividing the domain into smaller blocks, the possible improvement, and the additional calculations required.

Example: Consider a 100x20 metre block with 500 tonnes of earth to be moved to an adjacent block. Also consider a partitioning of the block into 50x10 sub-blocks (see Figure 7).

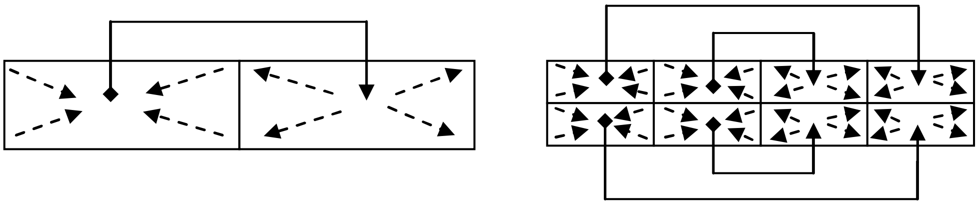


Figure 7. Graphical description of example

For *nx* = 1000 and *ny* =1000 the calculations are as follows:

Case 100x20: TKM_1 = 100 V + 2 ***TKM**(V,100,20,nx,ny)= 50000+ 2*13023.1 = 76046.2

Case 50x10: TKM_2 = 2*[150*(V/4) + 2***TKM**(V/4,50,10,nx,ny)]
+ 2*[50*(V/4) + 2***TKM**(V/4,50,10,nx,ny)]
= 2*[18750+2*1627.89] + 2*[6250+2*1627.89] = 63023.12

The results differ quite considerably in favour of the 50x10 partitioning.

6. Conclusions

The earthworks in a road construction case study were considered in this application paper. The main focus was to construct an earthwork allocation plan of least cost (or with least emissions), where cost is a direct measure of fuel consumption. The project site was partitioned into blocks and two block models were applied. 2D and 3D versions of the problem were compared to explore the effect of past simplifications. The computational burden of solving the block models (in 3D) was also investigated. In certain situations we have found that a 3D problem can be solved as a 2D one. When project domains (i.e. sites) are rectangular then it is most efficient to haul material along an axis rather than across it. In these situations, a 2D variant of the problem can be solved and replicated to obtain the optimal for the whole 3D problem. However in other scenarios this produces inferior solutions. As the number of blocks increases, the computational burden increases. This is particularly noticeable when considering 3D problems. In this paper we were unable to solve 3D problems with more than 2500 blocks, and a block size of less than 50 metres in length on a 7 km road project. More advanced solution techniques (such as graph theoretic, constructive algorithms, meta-heuristics) are hence necessary and/or the use of more powerful computers.

Reducing the block size and modelling more blocks is a superior approach as i) more earth movements are included and ii) earth can be moved to more destinations (i.e. improved decision making flexibility). Given the theory in Section 5, it is not possible to directly compare the solutions for different block sizes as the block size also affects the amount of “internal movements-haulages” that are included in modelling activities.

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