Finite Element Deletion and Topology Optimisation for Building Structural Optimisation

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Abstract

Buildings would not exist without a structural design. Although mechanical, electrical, and computer facilities now often put a much higher burden on the financial costs of a building than the structural design -related to both engineering and construction costs-, a building structure is unique in the sense that structural issues cannot be permitted, as they are life-threatening. Thus a building structure should primarily be safe, but nevertheless also economical. To help structural engineers with these goals, computer-assisted methods exist to determine the stress distribution in structural designs (e.g. the finite element method) and to optimise design components (e.g. topology optimisation). However, research on the optimisation of (complete) building structures is still relatively rare. In this paper, two methods for structural design optimisation have been compared for the application to complete building structural designs. This via a so-called research engine, in which spatial designs are transformed in structural designs, and vice versa, to investigate preliminary design processes. The two methods are compared for their effectiveness of optimisation, which shows that the method topology optimisation is more effective than the method element deletion, and if structural optimisation is used for exploring a solution space and evaluating the design process outcomes, this is an important conclusion. Besides, during topology optimisation a structural design remains stable, whereas element deletion may render the design unstable. However, when structural optimisation is used to study the primarily design process (e.g. via the research engine), the qualitative effects of both methods can be compared, and element deletion is computationally more efficient. Because even an unstable design will be usable in the research engine, for this case the method of element deletion is preferred.

Keywords: Building Structural Design, Topology Optimisation, Primarily Design Process, Finite Element Method, Finite Element Deletion

1. Introduction

Building (structural) engineers are used to the fact that design solutions are a product of a creative process: Not by working from a single problem towards a single solution, but by a very complex exploration of problems, requirements, and solutions simultaneously, as

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illustrated by a model and case studies by Maher (2000). The complexity of this type of design processes has led to three fields of research, related to dedicated computer tools that support the designer to explore a solution space and evaluate the design process outcomes, as described below.

The first field of research develops data models, which describe data and their relationships regarding the design process. Data models have been designed specifically for spatial design, e.g. by Björk (1992) and Borrmann and Rank (2009), and for structural design, e.g. by Weise *et al* (2000). For the interaction between spatial and structural design, Sause *et al* (1992) present an object-oriented approach to unify structural product and process models. Making use of the same object-oriented approach, Nguyen *et al* (1996) developed a concept for a data model, including a prototype program, for architectural design, structural design, and code compliance checking. Similar research was carried out by Khemlani *et al* (1998) and their most important achievement was the explicit formulation of the "space-structure dilemma" and a possible solution they proposed was a so-called "split-edge data structure" concept. Another proposition for a data model including structural and spatial information can be seen in the work of Matthews *et al* (1998). Eastman and Jeng (1999) took into account that the necessary modification of data models during the design process requires a specific data model set-up. They demonstrated this by a spatial, structural, and physical view of a building design example. Then Rivard and Fenves (2000) proposed a data model slightly later that incorporates both an object-oriented data model, and two design evolutionary capable abstraction levels for multiple views, again illustrated by a spatial-structural example. Mora *et al* (2006) worked out a very detailed data model, explicitly for spatial and structural design aspects, which was loosely based on the work of Rivard and Fenves (2000) mentioned above. This finally led, by the same authors, to an advanced design system prototype by Mora *et al* (2008).

The second field of research develops methods for actually generating spatial or structural design solutions for buildings. For spatial design, space-allocation, shape grammars, e.g Kotosopoulos (2005), and related methods, e.g. Oxman (1997), have been developed. For structural design, research has been carried on methods that actually generate a structural design, e.g. Rafiq and MacLeod (1988), Maher (1985), and Shaw *et al* (2008). Probably spanning the largest group of supporting tools within the field, finite element programs should be mentioned, which allow a detailed analyses of the stress distribution in a structural design, Zienkiewicz and Taylor (1988). Often using these finite element programs, also an enormous amount of research exists that optimises an existing structural design (component) by means of several possible optimisation methods, an overview is given by Kicinger (2005). It is remarkable that these optimisations methods are often used on two-dimensional problems regarding only components of a structural design. Literature in which three-dimensional problems are presented, incorporating a complete building structural design, can be found, Rafiq et al (2003), but only occasionally. This is the background for one of the contributions of this paper: Investigating the use of optimisation methods applied to complete building structural designs.

Most of the research projects mentioned above assume, explicitly or implicitly, that after the design of a preliminary spatial design, a preliminary structural design is developed, and this more or less subsequently. This does not correspond to the idea of exploring problems and solutions simultaneously, with which this section was started. However, a third field or research exists, that addresses exactly this idea: a strong interaction between disciplines, e.g. Maher (2000), Haymaker *et al* (2004). Inspired by this third field of research, a so-called research engine is under development, which develops and modifies a spatial-structural design through a number of cycles *N,* with each cycle numbered from *n* =1 to *N*, Hofmeyer (2007), figure 1. More specifically, a cycle consists of four steps: (1) a transformation from a spatial design (2*n*-1 in figure 1 on the right) to a structural design (2*n*-1); (2) the optimisation of the structural design (2*n*-1), which results in a new structural design (2*n*); and step (3) and (4), which interpret the new structural design as a spatial design (2*n*) and modify this spatial design such that it complies with the design requirements again. Hereafter, the cycle may be repeated, with integer *n* increased by 1.

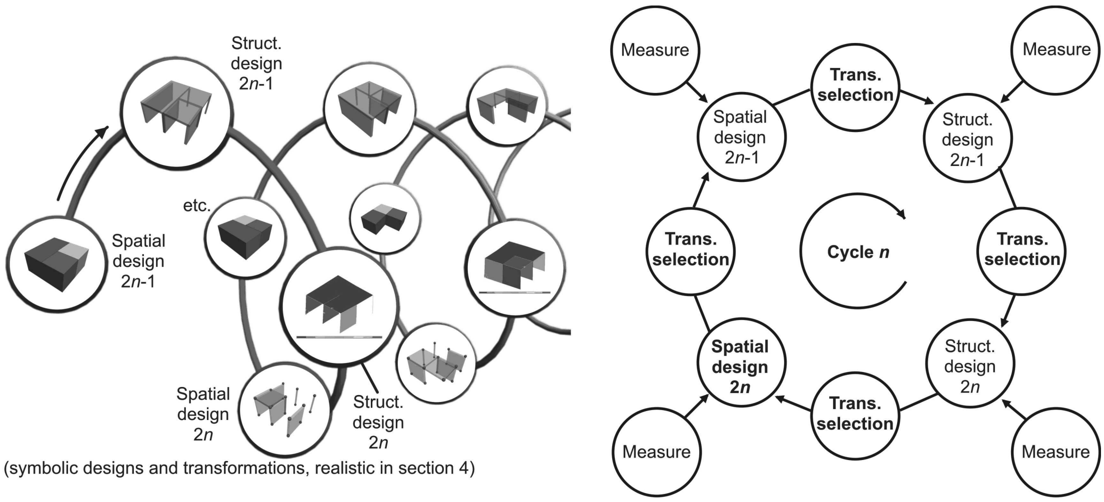

Figure 1: Research engine, symbolic on the left, schematic on the right

Several useful applications and interpretations exist for the research engine, Hofmeyer and Kerstens (2008), and in this paper it is used as a framework to study two possible methods for the structural optimisation of complete building structural designs. The first optimisation method, (finite) element deletion, has been specially developed for the research engine and its non-complex set-up fits the primarily design character of the engine. The second method, topology optimisation, is a frequently used, formal and more complex method, mostly used for two-dimensional problems related to structural design components, Bendsøe (1995).

After this introduction, in section 2 the element deletion method is explained, but this is only possible with a brief presentation of the research engine. Then, in section 3 the principles of topology optimisation are elaborated. Section 4 presents case studies using both optimisation methods; thereafter conclusions can be presented in section 5.

2. Element deletion

As the method of element deletion has been developed with the research engine in mind, and the research engine is used as framework in this paper, first the research engine will be presented here, although very briefly and only for the relevant parts as shown in figure 2. References will be given to allow for a more elaborated description.

The research engine is initiated for its first cycle (n = 1) with the input of a spatial design $2n$-1, which consists of rectangular spaces. Zoning, the first process in figure 2, then searches for all possible zones, which are rectangular sets that consist of one or more spaces. Then all possible solutions for constructing the spatial design out of these zones are found, see for more details (also on a possible combinatorial explosion) Hofmeyer and Bakker (2008). Formulated differently, the spatial design is redefined from consisting of spaces to consisting of (larger) zones, which is believed to resemble the working method of a structural engineer: Searching first for geometrical information (gridlines, openings, etc) on the larger scale of a structural design.

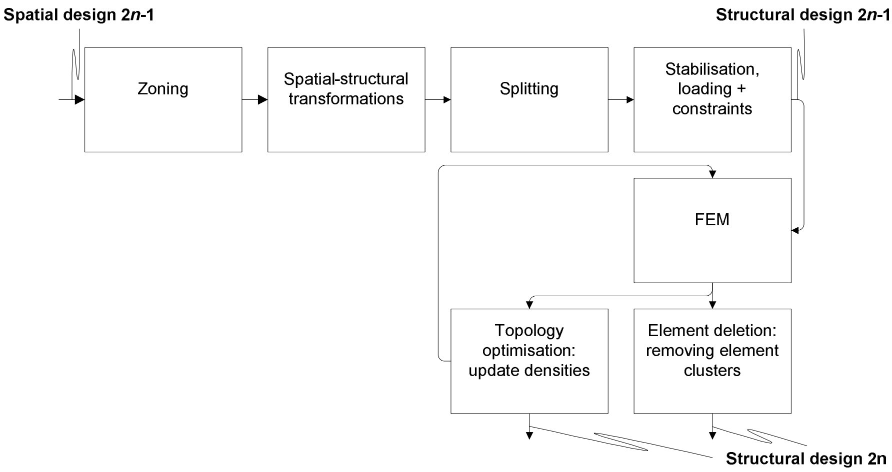

Figure 2: Research engine, process model of first two transformation steps

Hereafter, the second process in figure 2 applies spatial-structural transformation rules to the zoned spatial design. This implies that for every zone, depending on its geometrical properties, some structural elements -like shear walls, slabs, and columns- are added, Hofmeyer and Bakker (2008). For the resulting structural design two issues exist. First of all, the design is possibly not conformal, which means that once it is meshed using the finite element method, it may result in finite element nodes which are not connected, figure 3.

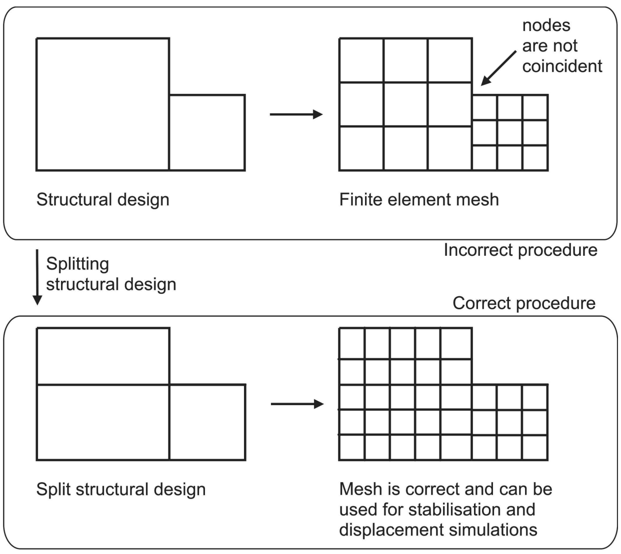

Figure 3: Structural design should be split to assure a correct finite element model

Therefore, the third process in figure 2, splitting, makes the structural design conformal, Hofmeyer *et al*, 2011. Secondly, due to the fact that structural elements are simply added to the zoned spatial design, without any further considerations, the resulting structural design is not necessarily stable. This is solved by using a dedicated stabilisation process, shown in figure 2 as the fourth process. During this stabilisation process, carried out by means of a finite element method, structural elements are added to the structural design until the structural stiffness matrix (used in the finite element method) is regular, Smulders and Hofmeyer (2012). Besides, this process also adds constraints (like boundary conditions simulating the foundation) and loads, after which structural design 2*n*-1 has been developed, as shown on the top right in figure 2.

The first step of the element deletion method is a simulation by the finite element method, as shown by process "FEM" in figure 2. The finite element method is based on approximating the complex displacement field over a continuous structure (e.g. a shear wall or slab) by a simple displacement field for a small part of the structure, an element. Relating all element displacement fields yields a system of linear equations, which once solved, results in an approximation of the displacements, stresses, strains, and strain energies in the structural design. When specifically the total strain energies of the elements are observed, naturally the elements will differ in their energy values. Given this fact, if a structural design should be made more efficient, it is then suggested to remove elements which show very low energy. This because if a structural element has low total strain energy, it either bears low forces (a quadratic relation exists between forces and energy) or shows high deformations (with a linear relation) and does not contribute significantly to the distribution of the applied loads.

For the method of element deletion, three aspects should be discussed. In the first place, for every optimisation method, it is the load case that almost completely determines the outcome of the optimised structural design. Because a structural design should not be optimised for e.g. a single wind direction -making it completely non-optimised in another direction-, several load cases have to be used. For each load case the finite element model is applied, where after for each element, the maximum total strain energy value among the load cases is selected for further processing. The load cases used are shown in figure 4 on the left. On the right, the load cases for topology optimisation are shown, to be presented in section 3. Further details on the selection of the specific load cases can be found in Hofmeyer and Davila Delgado (2012).

The second aspect concerning element deletion is the question how the elements to be deleted should be selected. For this, the K-means algorithm or Lloyd's algorithm is helpful, MacQueen (1967). This method sorts the finite elements in a number of groups specified by the user. Each finite element is put in the group for which the mean value is closest to the finite element total strain value. As such, the method generates groups with elements with similar energy values. In the currently used research engine, it has been found that grouping the finite elements in eight clusters yields workable results, however, future work should include a rigorous parameter study on the number of clusters to be used. Then, the user can decide how many clusters are to be removed, and hereafter a new optimised structural design 2*n* has been developed, as shown in figure 2 on the bottom right.

Finally, the third aspect is that a structural design 2*n*, as developed by the method of element deletion, is possibly not stable anymore. This is the case because finite elements have been deleted on the basis of their total strain energy level only, without taking into account any other argument, including that of stability of the structural design. It should be noted that for structural optimisation as a solely process this is indeed a problem. However, in the research engine structural design 2*n* is also a precursor for a new spatial design 2*n*, and this new spatial design can be developed from an unstable system without any problem. As such, for research engine applications, this latter drawback of element deletion does not exist.

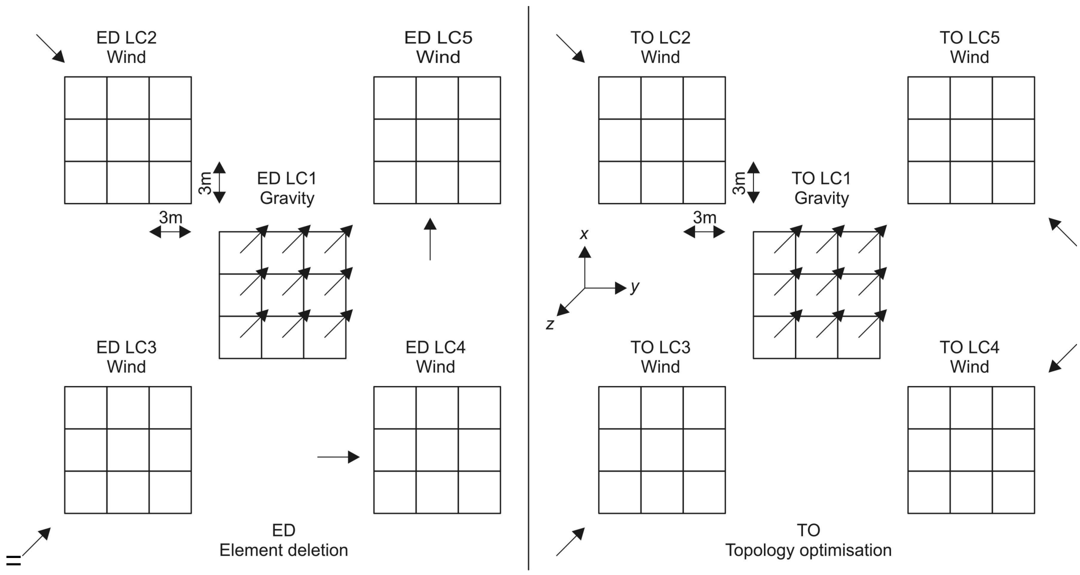

Figure 4: Load cases for element deletion on the left, for topology optimisation (in section 3) on the right

3. Topology optimisation

Topology optimisation is a more formal method, and due to its iterative character, presumably more effective than element deletion. For the topology optimisation method as presented in this paper, a method has been used exactly as described by Sigmund (2001). However, for completeness, a brief explanation will be given in this paper, with inevitably repeating some formulae. Just like for element deletion, also topology optimisation starts with a finite element simulation as shown in figure 2 on the right. Because topology optimisation is best explained using a formal notation, the finite element simulation will be formulated as:

$$\mathbf{KU} = \mathbf{F} \quad \text{Sigmund (2001)} \tag{1}$$

K is the structural stiffness matrix, **U** the displacement vector, and **F** is the force vector. Different from a normal finite element simulation, for topology optimisation **K** is assembled from element stiffness matrices $\mathbf{k}_e$ that are each multiplied with a so-called relative density. This relative density is a scalar x_e that can have a value between 0 to 1, and this makes it possible to lower or higher an element's stiffness. To enable calculating the sensitivity of an element to the objective to optimise, the relative density is powered by *p*, which is a value equal for all elements:

$$\forall_e \left\{ \mathbf{k}_e \rightarrow x_e^p \mathbf{k}_e \right\} \tag{2}$$

Topology optimisation is started with equation (1), with the same x_e for each element (for example 0.5). Hereafter, the objective to optimise is calculated, in this case the total strain energy:

$$c(\mathbf{x}) = \mathbf{U}^T \mathbf{K} \mathbf{U} = \sum_{e=1}^{N} x_e^p \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e \quad \text{Sigmund (2001)} \tag{3}$$

Assuming elements of the same size (volume), better x_e-values (leading to a lower objective) can be found as described by Bendsøe (1995):

$$x_e^{new} = \begin{cases} \max(x_{\min}, x_e - m) \text{ if } x_e \sqrt{\dfrac{-\partial c(x)/\partial x_e}{\lambda}} \leq \max(x_{\min}, x_e - m) \\ x_e \sqrt{\dfrac{-\partial c(x)/\partial x_e}{\lambda}} \text{ if } \max(x_{\min}, x_e - m) < x_e \sqrt{\dfrac{-\partial c(x)/\partial x_e}{\lambda}} \leq \min(1, x_e + m) \\ \min(1, x_e + m) \text{ if } \min(1, x_e + m) \leq x_e \sqrt{\dfrac{-\partial c(x)/\partial x_e}{\lambda}} \end{cases} \quad [35] \tag{4}$$

with *m* a constant with the aim to limit the maximum shift of the x_e -vector **x** and:

$$\partial c(x)/\partial x_e = -p x_e^{p-1} \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e \quad \text{Sigmund (2001)} \tag{5}$$

Using a bi-section method, the λ factor in equation (4) can be calculated such that the volume of the optimised structural design (the volume being a function of the new x_e vector **x**) is the same as during the initial run (with the initial x_e values equal to 0.5 in this case).

For certain reasons, as explained in Sigmund and Petersson (1998), it is useful to use a mesh-independency filter. This filter could be seen as partly averaging the element sensitivities over a certain domain (defined by value r_{min}). This filter modifies the values of equation (5) as follows:

$$\widehat{\frac{\partial c(x)}{\partial x_e}} = \frac{1}{x_e \sum_{f=1}^{N} \hat{H}_f} \sum_{f=1}^{N} \hat{H}_f x_f \frac{\partial c(x)}{\partial x_f} \quad \text{Sigmund (2001)} \tag{6}$$

with

$$\hat{H}_f = r_{\min} - dist(e, f), \quad \{ f \in N \mid dist(e, f) \leq r_{\min} \}, \quad e = 1, \ldots, N \quad \text{Sigmund (2001)} \tag{7}$$

dist(*e*,*f*) is equivalent to the distance between the centre points of (finite) element *e* and *f*, and r_{min}, being a user input, is the maximum distance between the element and other elements that should be considered.

Using the updated sensitivities of equation (6), a new prediction for the x_e-values can be made, where after the topology optimisation process is repeated until the change in x_e-values is sufficient small related to a user inputted threshold. This cycle is also shown in figure 2. And like the case for element deletion, also here a structural design 2*n* results.

4. Case studies

With the two optimisation methods presented above, case studies have been carried out, for which the results will be presented in this section. All case studies have been made fully automatically using the research engine as follows. The initial spatial design 2*n*-1 has a ground plan as shown in figure 4, with 3 × 3 spaces of each 3 × 3 meter, and a building height of 3, 21, or 60 meters, corresponding to 1, 7, or 20 levels with a level height equal to 3 m. The first process in figure 2, zoning, is instructed to develop a zoned spatial design by transforming each single space in a single zone. This seems to be the least advanced or interesting method of zoning, but to compare two methods of structural optimising, in this way a regular and fine grid of structural elements can be produced, and this in turn will enable the optimisation to reduce elements on the basis of a regular and fine grid as well. Therefore, spatial-structural transformations are used that add to each zone 4 shear walls and 1 slab on top of these shear walls. Due to the specific spatial design 2*n*-1, and the very specific settings for the zoning and spatial-structural transformations, the third process, splitting, is not necessary.

The fourth process, "Stabilisation, loading + constraints" is carried out completely, although stabilisation may not necessary due to the same reasons mentioned above: A regular and fine grid of shear walls and slabs is used, which makes the structural design stable naturally. Loading is applied as shown in figure 4. For element deletion, the five load cases are applied on the left. For topology optimisation, the load cases are applied as shown on the right, as explained in more detail by Hofmeyer and Davila Delgado (2013). For both methods, constraints are applied by fixing the lowest points of the structural design in all three independent directions *x*, *y*, and *z*, thus simulating a foundation.

Hereafter, one of the optimisation methods is applied as presented in section 2 or 3. In the finite element simulations, each structural element (i.e. a shear wall or slab) is meshed by 6 x 6 flat shell elements, which have a formulation as described in Batoz and Tahar (1982). After assembling the structural stiffness matrix, the system of equations is solved by BiCGSTAB, Eigen (2012).

Table 1 presents the case studies by their indentifying number and C value, which represents the number of finite element clusters (out of 8) that are removed. Figure 5 shows a typical run, in this case simulation A11, with from left to right in the top row: (a) spatial design 2*n*-1, (b) structural topology, and (c) structural design 2*n*-1 with clustered finite elements; every cluster is shown with a different grey. In the bottom row, from left to right: (d) optimised structural design 2*n*, (e) spatial design 2*n*-1 with the spaces to be removed coloured slightly more dark, and (f) spatial design 2*n*. The last two spatial designs are only shown here because they are produced automatically after structural design 2*n*-1, however, they will not be discussed in this paper.

Table 1. Case study identifier (left), number of element clusters removed (middle), and effectiveness of optimisation eo (right)

		Topology optimisation			**Element deletion**		
Low-rise (1 level)	LC1 Gravity load	A1	C3	2.27	A2	C3	1.45
	LC2 Wind load	A3	C2	2.27	A4	C1	1.88
	LC1,2,3,4,5 Combined loads	A5	C3	2.00	A6	C3	1.45
Mid-rise (7 levels)	LC1 Gravity load	A7	C2	3.03	A8	C2	1.23
	LC2 Wind load	A9	C2	4.76	A10	C2	2.27
	LC1,2,3,4,5 Combined loads	A11	C2	2.77	A12	C2	1.23
High-rise (20 levels)	LC1 Gravity load	A13	C2	3.57	A14	C2	1.27
	LC2 Wind load	A15	C2	6.67	A16	C2	1.30
	LC1,2,3,4,5 Combined loads	A17	C2	4.35	A18	C2	1.56

To compare the two different optimisation methods, each simulation of each case study is evaluated by using the following measure of effectiveness (*e*) of optimisation (*o*):

$$eo = \frac{E_{2n-1} * V_{FE;2n-1}}{E_{2n} * V_{FE;2n}} \quad (8)$$

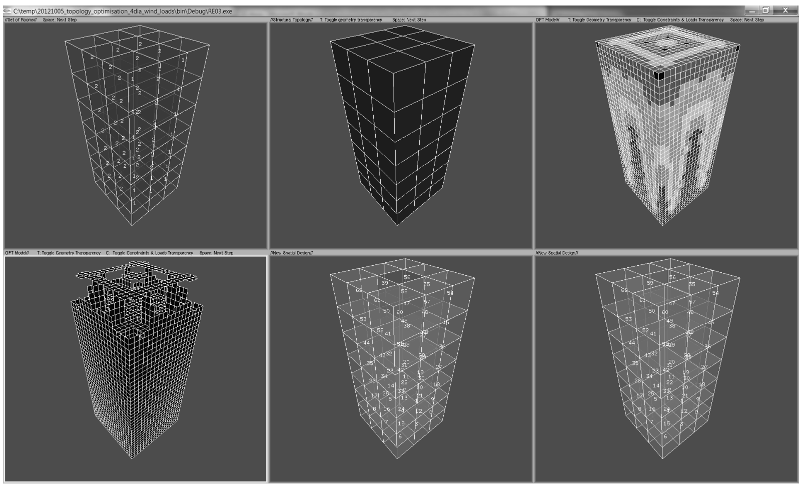

Figure 5: Research engine, typical run for case study A11

In equation (8), E stands for the sum of all finite elements' total strain energy, for structural design 2*n* and 2*n*-1 respectively, whereas V_{FE} is a variable that represents the total volume of finite elements used in the simulations, again for either design 2*n* or 2*n*-1, indicated by the subscript. Note that the measure of equation (8) is sensitive for both optimisation methods. If low strain elements are removed with the method element deletion, the total volume of finite elements will decrease for design 2*n*, whereas the total strain energy level will not change significantly, thus leading to a higher *eo* value, which indicates improved optimisation. For the topology optimisation method, the total volume of finite elements will be constant, however, here the total strain energy will decrease for design 2*n*, again increasing the value of the effectiveness of optimisation. More extended case studies, including more realistic building forms can be found in Hofmeyer & Davila Delgado (2013).

5. Conclusions

If table 1 is studied, it can be seen that topology optimisation is always more effective than element deletion, and even more significantly for taller designs. This can be explained by the fact that topology optimisation is an iterative procedure, whereas element deletion is only carried out once. Furthermore, a taller building implies more degrees of freedom in the finite element simulation, and topology optimisation benefits from higher numbers of elements.

If the specific load cases are taken into account, the case studies show that both methods perform better for LC2 Wind load than for other cases, in most situations. This is because a single wind load results in a strongly non-equally distributed strain energy field in the design, which enables a better optimised design more easily.

The effectiveness of optimisation measures shows that topology optimisation is more effective than element deletion, and if structural optimisation is used for exploring a solution space and evaluating the design process outcomes, this is a deciding factor. Besides, during topology optimisation a structural design remains stable, whereas element deletion may render the design unstable. However, when structural optimisation is used to study the primarily design process (e.g. via the research engine), the qualitative effects of both methods can be compared, and element deletion is computationally much more efficient. Because even an unstable design will be usable in the research engine, for this case the method of element deletion is preferred.

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